Budget Model

Introduction to PWBM Microsimulation with Application to Social Security

Presentation to Social Security Technical Panel, by Jagadeesh Gokhale

February, 2019
Figure 4: Social Security (OASDI) Projected Annual non-Interest Balance Ratios (percent)
The Social Security Trust Fund consists of excess revenues from Social Security taxes, which are invested in non-marketable Treasury securities. Interest income from these securities that is not used to pay benefits is also deposited in the Trust Fund.

The real (inflation-adjusted) value of the Trust Fund is expected to decline over time. Even during the next few years, when the difference between Social Security's interest income and its non-interest revenue shortfall will be positive, deposits into the Trust Fund will be insufficient to offset losses in its real value because of inflation. As the difference between program costs and income from Social Security taxes grows larger, interest income will eventually be insufficient to cover the non-interest revenue shortfall. Trust Fund securities would then have to be redeemed to pay lawful benefits, accelerating the decline in the Trust Fund's real value.

More information about the Penn Wharton Budget Model's Social Security simulator.

Policy Brief summarizing findings about Social Security’s Financial Condition.

More information about the ranges and default settings for the policy simulator dial controls.

NOTE: Constant dollars are for 2018 base year.
Motivating Microsimulation

Must be explicit:
- ... of assumptions (exogenous), equations (endogenous) and relationships.
- ... your view of the economy’s production side that generates the wage base.
- Can easily identify additional room for improvement of the model.

Therefore:
- Can back-test model on historical data to see how well functional relationships worked historically before projecting forward. We validate against 50+ parametric and non-parametric validations.
- Can easily generate uncertainty (e.g., confidence intervals) within a model consistent way without just letting things run loose or setting everything to “optimistic” or “pessimistic” in a non-model consistent manner (e.g., interest rates inconsistent with assumed growth).
“Future is Different than the Past”

Non-microsim actuarial approach:

- Some actuarial estimates done by age-sex (e.g., labor-force participation, mortality)
- Some estimates apply a growth number to entire labor force (productivity), SR vs. LR.

Example: Productivity

- Without microsim: Replacing a highly productive person going into retirement with a young person has no impact on tax base.
- With microsim: *Conditioning* on many attributes <age, gender, education, race, …>, productivity grows linear over time. Hence, *unconditional* productivity grows non-linear over time (see below).

Of course, one can add a time-indexed “add-factor” to the non-microsim actuarial model, but one needs to first run the microsim model to figure it out.
Let’s Dig In ...
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While education is a good proxy for income, future work will also condition on income and iterate between micro-sim and dynamic Gauss-Seidel style until convergence. Will discuss these in next slides in more detail as examples.
Step 1: Limited dependent regression on historical data (1996-2011) of female attributes per age (race, #children, education, marital status) to create 3x2x3x2=36 ratios of relative differences from mean, per age. Include time dummies intercepts to soak up unexplained variation by year. (Coefficients are not over-fitted, i.e., year independent.)


Step 3: Project forward probabilistically with time-varying gradient shift through 2040, using SSA OACT model (that is only conditional on age) or other time gradient source that allows for more conditioning.
Fitted Fertility Rates by Education:
#Kids=0, Married, Year=2015

Age

<High School

Some college

College or more

Black/Hispanic/Other

White/Asian

0.35

0.30

0.25

0.20

0.15

0.10

0.05

0.00

15 25 35 45

15 25 35 45

15 25 35 45
Fitted Fertility Rates by Education for #Kids=2+, Married, Year=2015

Age

<High School

Some college

College or more

Black/Hispanic/Other

White/Asian
Fitted Fertility Rates by Education for #Kids=0, Unmarried, Year=2015
Fitted Fertility Rates by Education for #Kids=0, Married, Years=1997, 2006, 2015
Microsimulation Example: Mortality (Simplified Explanation)

**Step 1:** Published death rates (NCHS tables, 1999-2012) by age, sex, and race are further decomposed by education and marital status using differentials published in the academic literature. This provides 50 ratios of relative differences that are applied to historical average mortality per age and sex.

**Step 2:** Start with 1996 population of individuals aged 0 – 120, simulate deaths probabilistically by attribute. Simulated CDF ~ population.

**Step 3:** Project forward probabilistically with time-varying gradient shift through 75 years, using SSA OACT model (that is only conditional on age and sex) or other gradient source that allows for more conditioning.
Projected Mortality Target

White Males and White Females
Example: Effect of distinguishing mortality rates by ethnicity

Mortality by age and gender - 2000

- SIM 2000 Female
- SIM 2000 Male
- SSA 2000 Female
- SSA 2000 Male

Mortality by age and gender - 2060

- SIM 2060 Female
- SIM 2060 Male
- SSA 2060 Female
- SSA 2060 Male

Mortality by age and gender - 2080

- SIM 2080 Female
- SIM 2080 Male
- SSA 2080 Female
- SSA 2080 Male

Mortality by age and gender - 2100

- SIM 2100 Female
- SIM 2100 Male
- SSA 2100 Female
- SSA 2100 Male
Race / Ethnicity (1996 – 2050)

Census Data

1996

Microsimulation

1996

Percent of total population

Age

White
Black
Hispanic
Asian
Other

White
Black
Hispanic
Asian
Other

20 30 40 50 60 70 80

0.000
0.005
0.010
0.015
0.020
0.025
# Individuals and Families: General Simulation Sequence

## Initial Population

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- **Initial population from ASEC with all attributes as of Dec. 31, 1996 aged 0-120**

## Aging

- **Add 1 year to each person’s age:** \( A_t = A_{t-1} + 1 \)

## Family Split-offs

- **Those who turn 18 split-off and form their own family units:** \( p_{\text{split}}(\text{age18}) = 1 \)

## Fertility

- **Females aged 14:49 \( \rightarrow \) new age-0 pop.:** \( f_t = f(\text{age}, \text{sex}, \text{ethn}, \text{educ}_t, \text{mar}_t, \#\text{kids}_t) \)

## Mortality

- **Death rates:** \( d_t = d(\text{age}_t, \text{sex}, \text{ethn}, \text{educ}_t, \text{mar}_t) \); \( d(\text{age120}) = 1 \)

## Education

- **Age 6+ advance education years:** \( p(\Delta e) = p(e_{t-1}|\text{age}_t, \text{ethn}, \text{gender}) \)

## Disability

- **People 0+ transit in-out of work impairment status (not SSDI):** \( \delta_t = d(\delta_{t-1}|\text{age}, \text{sex}, \text{ethn}) \)

## LFP and FTE Work Hours

- **People 18+ through FTE weeks employed (0-104):** \( \omega_t = \omega(\omega_{t-1}|\text{age}_t, \text{sex}, \text{ethn}, \ldots \text{many}) \)

## Employment

- **Those not working may be Unemployed:** \( u_t = u(\omega_t = 0|\text{age}_t, \text{sex}, \text{ethn}, \ldots \text{many}) \)

## Immigration

- **Immigrants aged 0:119 (all attributes):** \( I^x_t = I^x_t x P_t \) \( \{i: \text{immig rate}; S: \text{legal/undoc}; P: \text{pop}\} \)

## Divorce

- **Divorce: Immediate entry into marriage market:** \( m_t = m(m|\text{age}_t, \text{sex}, \text{ethn}, \text{educ}_t) \)

## Marriage

- **Marriage market (age 18+):** \( v_t = v(s|\text{age}_t, \text{sex}, \text{ethn}, \text{educ}_t) \)

## Calculators

- **Wages and S. E. Income**
  - Capital assigner (calibrated to BLS estimates of capital services)
  - Benefits calculators: Social Security (other transfers under development)
  - Tax calculators (Individual Income, Payroll, and Corporate)

## Dynamic

- **GE-OLG Model with heterogeneous agents – attributes calibrated from SIM**

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**Discussed in more detail on PWBM website. Will just review model outputs for now.**
Immigrant Status (1996 – 2050)

Census Data

Microsimulation

1996

Percent of total population

0.000 0.005 0.010 0.015 0.020 0.025

Age

20 30 40 50 60 70 80

Immigrant
Non-immigrant

1996

Percent of total population

0.000 0.005 0.010 0.015 0.020 0.025

Age

20 30 40 50 60 70 80

Immigrant
Non-immigrant
Disability (1996 – 2070)

### Census Data

#### 1996

- Non-disabled
- Disabled

### Microsimulation

#### 1996

- Non-disabled
- Disabled

---

Age

Percent of total population
Wage income deciles (1996 – 2070)

Census Data

Microsimulation

1996

0-10th decile
10-20th decile
20-30th decile
30-40th decile
40-50th decile
50-60th decile
60-70th decile
70-80th decile
80-90th decile

Percent of total population

Age

1996

0-10th decile
10-20th decile
20-30th decile
30-40th decile
40-50th decile
50-60th decile
60-70th decile
70-80th decile
80-90th decile

Percent of total population

Age
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### Relationships

Come from a structural dynamic programming model, given their importance.
Microsimulation: Marriage (Very Brief)

Structural DP model of marriage and divorce (Sophie Shin dissertation)

Existing micro-datasets on *new marriages* are inadequate – use stocks

Person types by initial age and 15 (5 race x 3 education) categories \{a, r, e\}

Measure annual change in the stocks of \{a, r, e\} marriages and subtract marriage dissolutions from divorce and death

Simulation: Marriages structured in two stages – Meeting and Acceptance

- Meeting rates: \( x \) percent within race, \((1-x)\) percent cross-race (including same race)

- Acceptance rates based on match quality – multiplicative weight on comparable age and education levels

\( \rightarrow \) assortative pairing by age and education as observed in micro-data
Assortative Mating (1996 – 2050)

Census Data

Microsimulation

1996

Asian + Asian
Asian + Black
Asian + Hispanic
Asian + Other
Asian + Unmarried
Asian + White
Black + Asian
Black + Black
Black + Hispanic
Black + Other
Black + Unmarried
Black + White
Hispanic + Asian
Hispanic + Black
Hispanic + Hispanic
Hispanic + Other
Hispanic + Unmarried
Hispanic + White
Other + Asian
Other + Black
Other + Hispanic
Other + Other
Other + Unmarried
Other + White
White + Asian
White + Black

Percent of total population

Age

0.000
0.005
0.010
0.015
0.020
0.025

20 30 40 50 60 70 80
Marriage (1996 – 2070)

Census Data

1996

- White
- Black
- Hispanic
- Asian
- Other
- Unmarried

Percent of total population

Age

Microsimulation

1996

- White
- Black
- Hispanic
- Asian
- Other
- Unmarried

Percent of total population

Age
Family Composition (1996 – 2070)

Census Data

Microsimulation

1996

Percent of total population

20 30 40 50 60 70 80

Age

Non-family
Single-headed family
Dual-headed family

1996

Percent of total population

20 30 40 50 60 70 80

Age

Non-family
Single-headed family
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*Briefly discussed next (and Appendix)*
Labor Force: FTE Weeks worked

A. Determine FTE weeks employed (census micro-data; up to 104 per year)
   - Initial FTE weeks regression: \( \omega_t = f(age_t, gender, ethnicity, \ldots) \)
     - If \( \omega_t = 0 \), set \( ec_t = 0 \) … employment class = “not working”
   - \( ec \) Transition (0/1/2/3): \( ec_t = Tec(ec_{t-1}|age_t, gender, ethnicity, \ldots) \)
   - Set FTE weeks: \( w_k = w(w_{k-1}, ec_t|age_t, gender, ethnicity, \ldots), k > t \)

B. Regardless of the outcome for A, determine annual weeks unemployed
   - \( u_t = u(age_t, gender, ethnicity, \ldots) = \) calendar weeks looking for work adjusted for cap on total weeks
Annual Work-weeks and Looking-for-Work weeks
Labor Force Profiles

### Census Data

Population Shares by Age, Gender, and Ethnicity - 1998

### Microsimulation

Population Shares by Age, Gender, and Ethnicity - 1998
Labor Force Profiles

Census Data

Microsimulation

Average Weeks Worked by Age, Gender, and Ethnicity - 1996

Average Weeks Worked by Age, Gender, and Ethnicity - 1996
Aggregation: Capital, Efficiency-Adjusted Labor Services 
GDP, Labor Share and the Wage Base

Cobb-Douglas production function framework: Nominal GDP

\[ Y_t = P_t A_t K_t^\alpha L_t^{1-\alpha} \]

\( Y_t \) = Total output  
\( P_t \) = Price level  
\( A_t \) = Multifactor productivity  
\( K_t \) = Capital services  
\( L_t \) = Efficiency adjusted labor services  
\( \alpha \) = Output elasticity of capital

See Appendix and PWBM website for estimation process
Labor Earnings

All worker characteristics determine efficiency at work per period (year)

- Age, gender, ethnicity, marital status, family size, FTE weeks/year, Unemployment weeks/year, health impairment, primary/secondary earner status, education years, birth-year, immigrant status, legal status, years since immigrated…with interactions

AR(1) regression on pooled cross-year data (see Appendix)

- Isolates contribution of each worker attribute to productivity in the workplace
- Regression parameters can be applied to historical attributes and aggregated → GDP
- Can also be applied to projected worker attributes to project productivity according to distributions of worker attributes in future years
Productivity changes from compositional effects
Population Aging – Worker Population Split by 1968 Birth-year - 1

Worker Population Share

- 1968 or earlier Birth
- 1969 or Later Birth

Average Productivity

- 1968 or Earlier Birth
- 1969 or Later Birth

Net Productivity Effect

Percent Change from 2018

-3.0 -2.0 -1.0 0.0 1.0 2.0 3.0
2018 2023 2028 2033 2038 2043 2048
Worker Population by College or More - 2

Net Productivity Effect

Percent change from 2018

2018 2023 2028 2033 2038 2043 2048

Net Productivity Effect
Worker Population by Annual Weeks Worked - 1

Worker Population Shares

Average Productivity

Constant Dollars per FTE Week

2018 2023 2028 2033 2038 2043 2048

2018 2023 2028 2033 2038 2043 2048

1-9 10-26 27-52 53-75 76-104
Worker Population by Ethnicity - 1

Worker Population Shares

- White
- Hispanic
- Asian
- Other
- Black

Average Productivity

- Black
- White
- Hispanic
- Asian
- Other

Constant Dollars per FTE Week

2018 2023 2028 2033 2038 2043 2048
Worker Population by Ethnicity - 2

Net Productivity Effect

Percent Change From 2018

2018 2023 2028 2033 2038 2043 2048

2.00 1.50 1.00 0.50 0.00 -0.50 -1.00 -1.50 -2.00
Worker Population by Gender - 1

Worker Population Share

- Male
- Female

Average Productivity

- Male
- Female

Constant Dollars per FTE Week
Worker Population by Gender - 2

Net Productivity Effect

Percent Change from 2018

2018 2023 2028 2033 2038 2043 2048
Worker Population by Legal Status - 1

Worker Population Shares by Legal Status

Average Productivity by Legal Status

- Undocumented
- Legal
- Native
- Naturalized
Worker Population by Legal Status - 2

Net Productivity Effect

Percent Change from 2018

Net Productivity Effect
### Labor Productivity: Historical Averages

<table>
<thead>
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<th>Source</th>
<th>Labor productivity average 1997-2017</th>
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<tr>
<td>Census ASEC (data)</td>
<td>1.46</td>
</tr>
<tr>
<td>PWBM (back test)</td>
<td>1.40</td>
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<tr>
<td>SSA (data)</td>
<td>1.64</td>
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Main difference between Census (data) and SSA (data) due to differences in construction of L term in productivity (Y/L). PWBM focuses on Census construction since we focus on a broader set of fiscal policies.
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<td>Those who turn 18 split-off and form their own family units: ( p_{\text{split}}(\text{age18}) = 1 )</td>
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<tr>
<td>3</td>
<td>Fertility</td>
</tr>
<tr>
<td></td>
<td>Females aged 14:49 → new age-0 pop. ( f_t = f(\text{age}_t, \text{ethn}, \text{educ}_t, \text{mar}_t, #\text{kids}_t) )</td>
</tr>
<tr>
<td>4</td>
<td>Mortality</td>
</tr>
<tr>
<td></td>
<td>Death rates: ( d_t = d(\text{age}_t, \text{sex}, \text{ethn}, \text{educ}_t, \text{mar}_t); \ d(\text{age120}) = 1 )</td>
</tr>
<tr>
<td>5</td>
<td>Education</td>
</tr>
<tr>
<td></td>
<td>Age 6+ advance education years: ( p(\Delta e) = p(e_{t-1}</td>
</tr>
<tr>
<td>6</td>
<td>Disability</td>
</tr>
<tr>
<td></td>
<td>People 0+ transit in-out of work impairment status (not SSDI): ( \delta_t = d(\delta_{t-1}</td>
</tr>
<tr>
<td>7</td>
<td>LFP and FTE work hours</td>
</tr>
<tr>
<td></td>
<td>People 18+ through FTE weeks employed (0-104): ( \omega_t = \omega(\omega_{t-1}</td>
</tr>
<tr>
<td>8</td>
<td>Employment</td>
</tr>
<tr>
<td></td>
<td>Those not working may be Unemployed: ( u_t = u(\omega_t = 0</td>
</tr>
<tr>
<td>9</td>
<td>Immigration</td>
</tr>
<tr>
<td></td>
<td>Immigrants aged 0:119 (all attributes): ( I_t = I_t^{\text{immig}}xP_t {i: \text{immig rate; S:legal/undoc; P: pop}} )</td>
</tr>
<tr>
<td>10</td>
<td>Divorce</td>
</tr>
<tr>
<td></td>
<td>Divorce: Immediate entry into marriage market: ( m_t = m(m</td>
</tr>
<tr>
<td>11</td>
<td>Marriage</td>
</tr>
<tr>
<td></td>
<td>Marriage market (age 18+): ( v_t = v(s</td>
</tr>
<tr>
<td>12</td>
<td>Calculators</td>
</tr>
<tr>
<td></td>
<td>Wages and S. E. income</td>
</tr>
<tr>
<td></td>
<td>Capital assigner (calibrated to BLS estimates of capital services)</td>
</tr>
<tr>
<td></td>
<td>Benefits calculators: Social Security (other transfers under development)</td>
</tr>
<tr>
<td></td>
<td>Tax calculators (Individual Income, Payroll, and Corporate)</td>
</tr>
<tr>
<td>13</td>
<td>Dynamic</td>
</tr>
<tr>
<td></td>
<td>GE-OLG Model with heterogeneous agents – attributes calibrated from SIM</td>
</tr>
</tbody>
</table>
Benefits calculator
- Very detailed rules (even the rounding / truncation rules)
- Validated using 20,000+ different individual & household types and compared against OACT FORTRAN. All must be within one penny.

Tax calculator can be run on multiple bases:
- “Static” (no elasticities)
- “Conventional”
  - Business entity type elasticity (when appropriate)
  - Income deferral elasticity (when appropriate)
- “Dynamic” (with OLG model)
  - Labor supply elasticity to net tax (PVB - PVT at margin)
  - Allows for GDP / tax base growth effects
Macro-Model Integration
Integrated Micro-simulation and Stochastic OLG model

Key inputs reflect the detailed demographic and policy heterogeneity from the microsimulation model
• Social Security, individual and business income, taxes, transfers, etc.

OLG model single firm, multi-agent Bewley-type model, with government
• Heterogeneity – age, productivity, wealth, immigrant status, earnings
• Government – taxes, transfers, “unproductive spending,” and debt

Baseline and policy runs: Equilibrium time paths of household and firm decision rules and macro time-series
Figure 4: Social Security (OASDI) Projected Annual non-Interest Balance Ratios (percent)
Appendix
Cobb-Douglas production function framework: Nominal GDP

\[ Y_t = P_t A_t K_t^\alpha L_t^{1-\alpha} \]

- \( Y_t \) = Total output
- \( P_t \) = Price level
- \( A_t \) = Multifactor productivity
- \( L_t \) = Capital services
- \( L_t \) = Efficiency adjusted labor services
- \( \alpha \) = Output elasticity of capital

Rewrite:

\[ \ln Y_t - \ln P_t - \ln A_t - \alpha \ln K_t = (1 - \alpha) \ln L_t \]

\( P, A, \) and \( \alpha \) are specified exogenously, while \( K \) and \( L \) are determined within the simulation…How?

Nominal Compensation Share:

\[ W_t \equiv (1 - \alpha)Y_t = (1 - \alpha)P_t A_t K_t^\alpha L_t^{1-\alpha} \]

Rewrite and adjust for prices, MFP, and capital deepening:

\[ \ln Z_t = \ln W_t - \ln P_t - \ln A_t - \alpha \ln K_t = \ln (1 - \alpha) + (1 - \alpha) \ln L_t \]

Efficiency adjusted labor input is modeled as

\[ L_t = \sum_i l_{it} = \sum_i e^{\sum_{j=1}^{m} \theta_j x_{jit}} \]

Cross-year stacked regression at worker level, annual freq.:

\[ \ln L_t = \frac{\ln z_{it} - \ln (1 - \alpha)}{(1 - \alpha)} = f(x_{it}; \theta) = \sum_{j=1}^{k} \theta_j x_{jit} + u_{it} \]
Regression executed on gross wages although labor share includes employee benefits

Wages are observed in micro-data, total compensation is not

Estimate and add non-wage compensation: social security employer taxes, pension and health insurance benefits (based on simulated coverages), and other compensation

- Social Security employer taxes are easy!
- Pension and health benefits assumed proportional to simulated wage; benchmarked to national totals
- Nonwage benefits benchmarked to national total by adjusting other benefits

Stacking observations from different years (1996-2016) - cross-year wages have been “placed on par” by removing the effect of inflation, MFP, and capital deepening

Assume all workers hired on spot-market - no long-term implicit contracts that cause current compensation to diverge from current productivity → Observed wage = worker's current productivity
Aggregation

\[ \tilde{L}_t = \sum_i \tilde{l}_{it} = \sum_i \ln \tilde{z}_{it} = f(\tilde{x}_{it}; \tilde{\theta}) = \sum_{j=1}^k \tilde{\theta}_j \tilde{x}_{jit} + \tilde{\mu}_{it} \]

- Apply regression coefficients (\(\tilde{\theta}\)) to simulated (\(\tilde{~}\)) worker attributes \(\rightarrow\)
  - Captures contribution of worker attributes to productivity – **core labor input**

- Shock term – segmented/tailored-bootstrap from distribution of regression errors
  - idiosyncratic shock - captures unexplained variability in worker productivity – assumed to be transitory
Observed work-choices and wage levels are not independent

- Wage regression on attributes w/o adjustment for selection
  - biased coefficient estimates

- Fixing the selection bias (adjust error term with Mills ratio) matters for estimating effects of worker attributes on potential productivity, that is, not conditioning on work choice

- But such a fix is not needed when simulating wages: coefficient estimates applied post selection of work choice estimated separately

Current procedure

- Simulate work choice and FTE hours from micro-data (described above)

- Labor hours enter as an attribute in the wage-regression $\rightarrow$ core wages, $lnL_t$
Conditional Productivity Differences by Specific Attribute at Selected Ages
Conditional Productivity Differences by Specific Attribute at Selected Ages
Conditional Productivity Differences by Specific Attribute at Selected Ages

**Unemployment Duration**
- Weeks Unemployed = 20
- Weeks unemployed = 0

**Legal Immigration Status**
- U.S. Native
- Naturalized Immigrant
- Legal Immigrant
- Unauthorized Immigrant

**Immigrant**
- Immigrant
- Non-immigrant

**Years Since Immigrated**
- 25
- 15
- 12
Labor Productivity Growth Decomposition
Output Growth Decomposition

The production function framework implies the output growth decomposition

\[
\frac{1}{Y_t} \frac{dY_t}{dt} = \frac{1}{A_t} \frac{dA_t}{dt} + \alpha \frac{1}{K_t} \frac{dK_t}{dt} + (1 - \alpha) \frac{1}{h_t} \frac{dh_t}{dt} + (1 - \alpha) \frac{1}{L_t} \frac{dL_t}{dt}
\]

Each component on the right-hand-side of the equation above contributes to total GDP growth (the term on the left-hand-side).

Labor productivity growth then equals

\[
\frac{1}{Y_t} \frac{dY_t}{dt} - \frac{1}{L_t} \frac{dL_t}{dt} = \frac{1}{A_t} \frac{dA_t}{dt} + \alpha \frac{1}{K_t} \frac{dK_t}{dt} + (1 - \alpha) \frac{1}{h_t} \frac{dh_t}{dt} - \alpha \frac{1}{L_t} \frac{dL_t}{dt}
\]

2018-27: 
\[
1.674 - 0.795 = 0.615 + 0.345*1.864 + 0.655*(-0.166) - 0.345*0.795 = 0.88
\]

2018-37: 
\[
1.494 - 0.570 = 0.640 + 0.345*1.753 + 0.655*(-0.247) - 0.345*0.624 = 0.87
\]

2018-92: 
\[
1.686 - 0.510 = 0.658 + 0.345*1.943 + 0.655*0.028 - 0.345*0.510 = 1.17
\]
Production Function: Multifactor Productivity

Multifactor Productivity ($A$) Growth Rate

BLS Private Business Sector Average Growth (1988-2017) ......................... 0.86
Convert to Total Economy Basis (OO-Housing/Government/Non-Profit) ..... 0.66

![Graph of Multi-Factor Productivity]

Projection Assumption: 0.658 percent/year
Production Function: Capital Share

Capital Services Share ($\alpha$)

BLS Total Economy estimate includes extra income items
Remove net income of Government/Non-Profit from capital income and find share of GDP……………………………………………………………………………………………………………………… 0.345
Production Function: Capital Services

Capital Services ($K$)

BLS: Industry specific investment history + depreciation \(\rightarrow\) current stock
Depreciation rates: Differential rates of service release by short- and long-lived capital
Investment rate: Capital service input assumed to grow with labor input – the United States is assumed to remain open to trade and capital flows

Capital Services ($K$) Growth

PWBM Projection Average (2018-92): 1.943 percent/year
Production Function: Employment growth

Worker head count based on work choices correlated with projected person attributes

PWBM microsimulation projections
Production Function: Labor efficiency growth

Worker productivities correlated with projected worker attributes conditional on work choice

PWBM microsimulation projection

PWBM Projection Average (2018-92): 0.028 percent/year
Production Function: Labor input growth
Efficiency adjusted labor services
PWBM microsimulation projection

![Graph showing labor input growth (h×L) with average growth (2018-92) 0.538 percent/year](Image)